Faculty of Mathematics and Physics, Charles University

Mathematical problems of prisoners

Sep 21 2015

Ondrej Škopek

Faculty of Mathematics and Physics, Charles University

Mathematical problems of prisoners and students

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About me

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<oskopek@matfyz.cz>

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<oskopek@matfyz.cz>
oskopek.com

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<oskopek@matfyz.cz>
oskopek.com

Ask questions

About me

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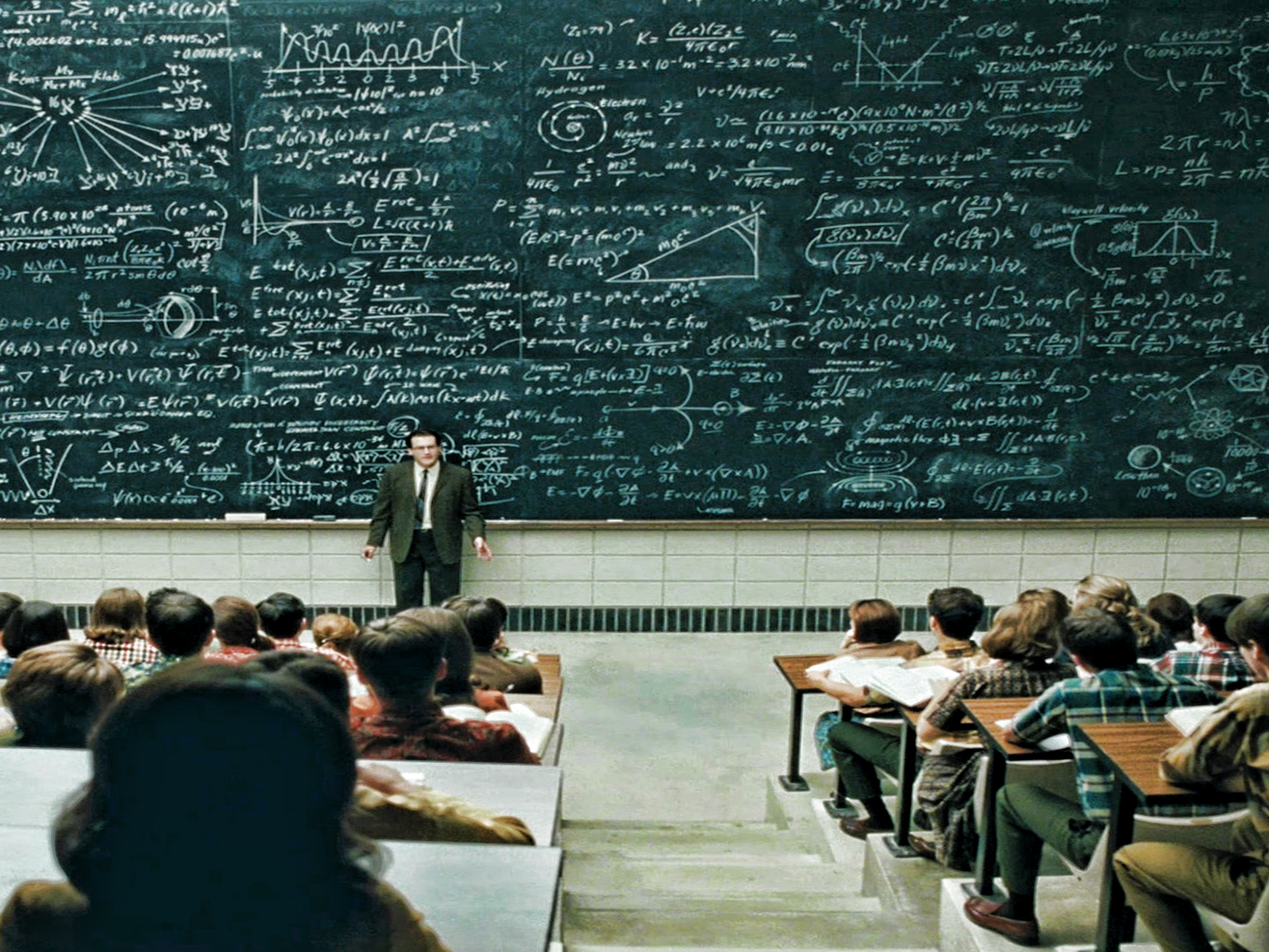
<oskopek@matfyz.cz>
oskopek.com

Ask questions

many and often

About you

What is mathematics?



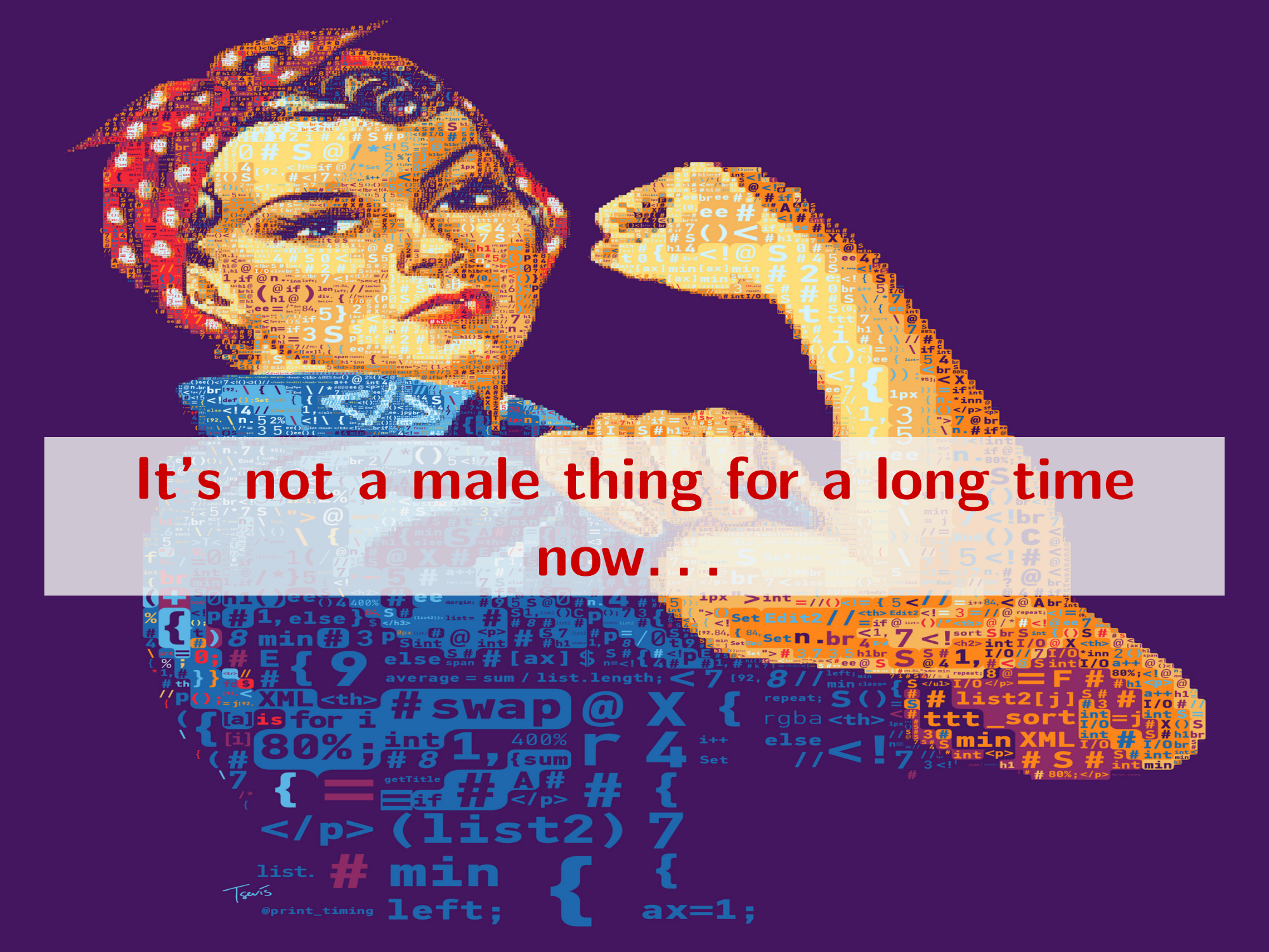

Recognize this feeling?


Groups
Homomorphisms
Polynomials
Definition
Question: What are the approximate polynomials?
Theorem
Inverse conjecture for Gowers norms
Conjecture


fiesta.cc
tumblr.
タンブラー。
venmo
P2PU
Aviary

Math in practice today


average = sum / list.length;
</p> (list2) 7
list. # min { {
left; { ax=1;
Tsevis
@print_timing


It’s not a male thing for a long time now...

“A mathematician is the only kind of scientist that can rightfully proclaim: I’ll lie on the couch, close my eyes and work.”

"A mathematician is the only kind of scientist that can rightfully proclaim: I'll lie on the couch, close my eyes and work."

– Keith Devlin

Number magic

Number magic

Surely you've seen a similar exercise before:

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5
- Multiply it by 2

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5
- Multiply it by 2
- Add 2

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5
- Multiply it by 2
- Add 2
- Multiply by 3

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5
- Multiply it by 2
- Add 2
- Multiply by 3
- Subtract the double of the original number

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5
- Multiply it by 2
- Add 2
- Multiply by 3
- Subtract the double of the original number
- Add 6

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5
- Multiply it by 2
- Add 2
- Multiply by 3
- Subtract the double of the original number
- Add 6
- Divide by 4

Number magic

Surely you've seen a similar exercise before:

- Think of a number between 1-5
- Multiply it by 2
- Add 2
- Multiply by 3
- Subtract the double of the original number
- Add 6
- Divide by 4
- Subtract the original number (again)

Number magic

Result:

Number magic

Result: 3

Number magic

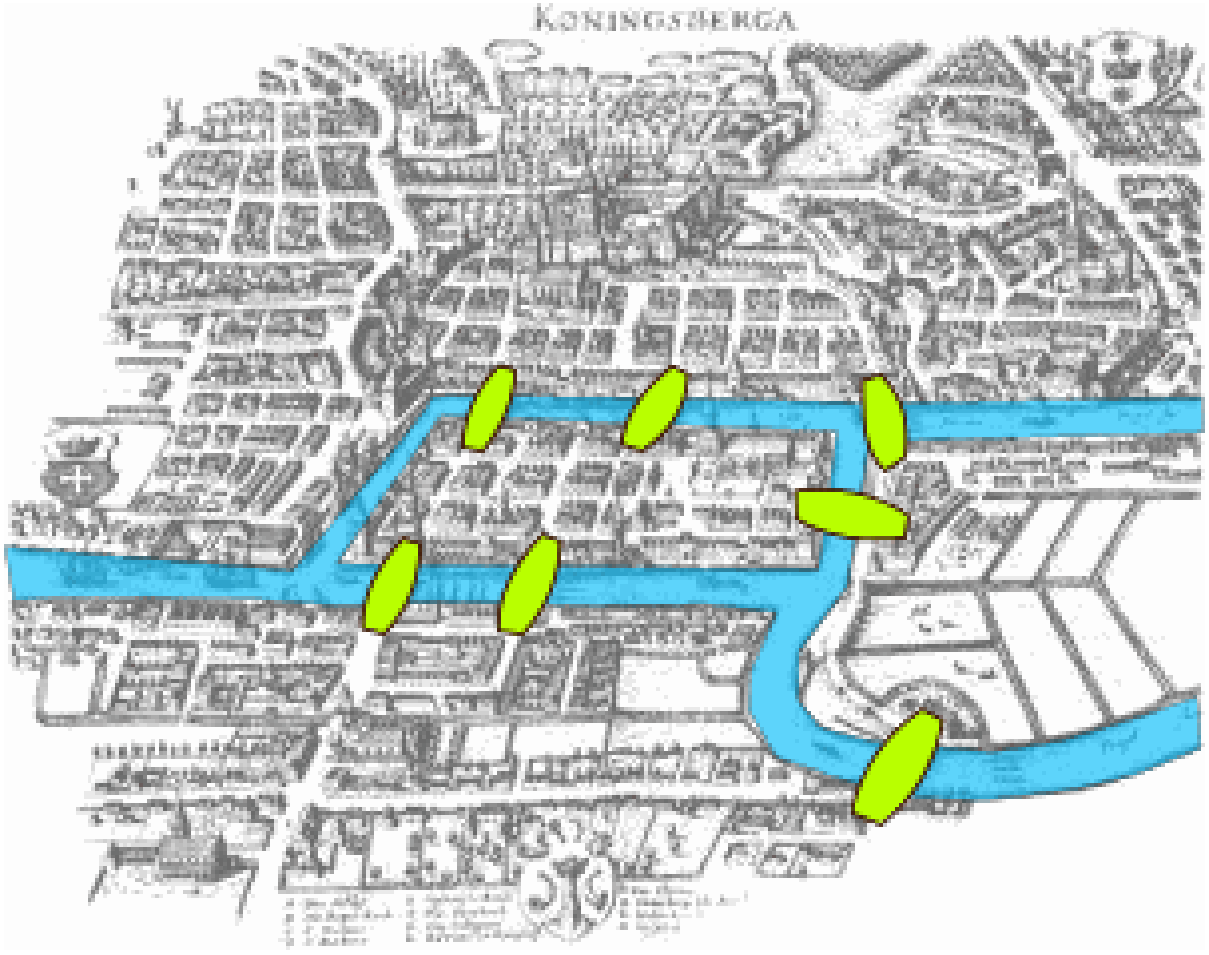
- Will you always get the same result? Or just for the numbers 1-5?

Number magic

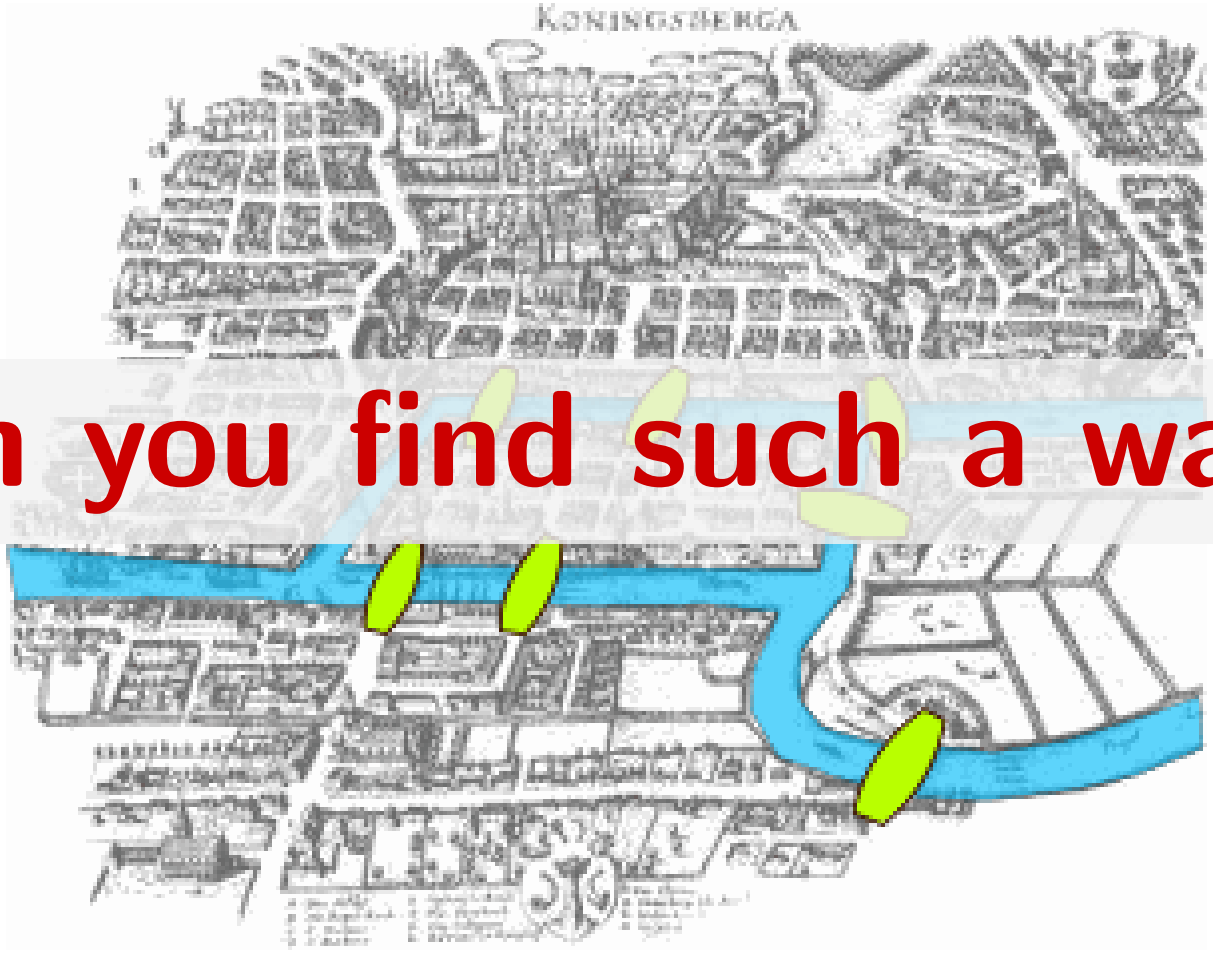
- Will you always get the same result? Or just for the numbers 1-5?
- Why doesn't it matter what number you start with?

Seven bridges of Königsberg

Seven bridges of Königsberg



Seven bridges of Königsberg



Can you find such a walk?

Seven bridges of Königsberg

- Why not?

Seven bridges of Königsberg

- Why not?
- Can it be shown, that such a walk doesn't exist?

Seven bridges of Königsberg

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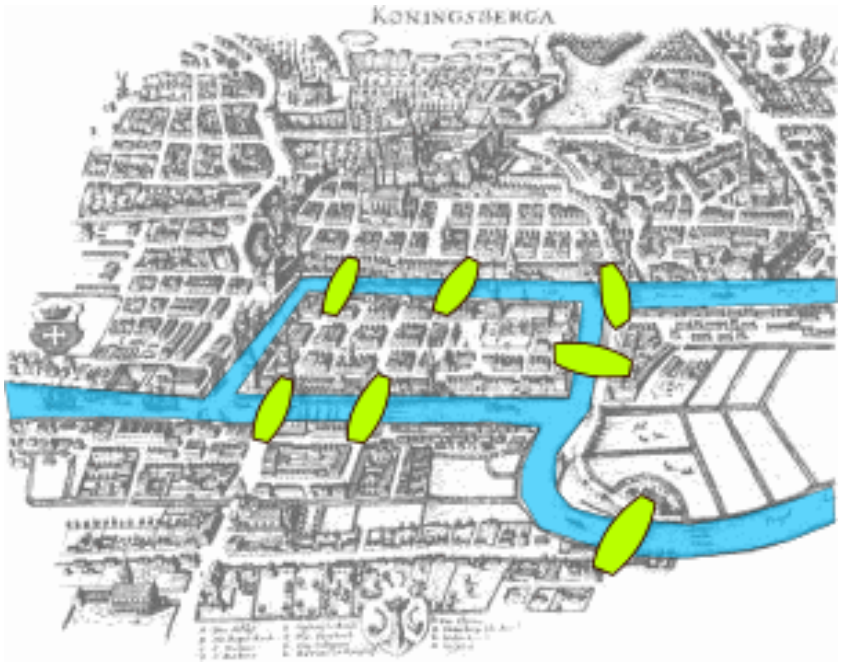
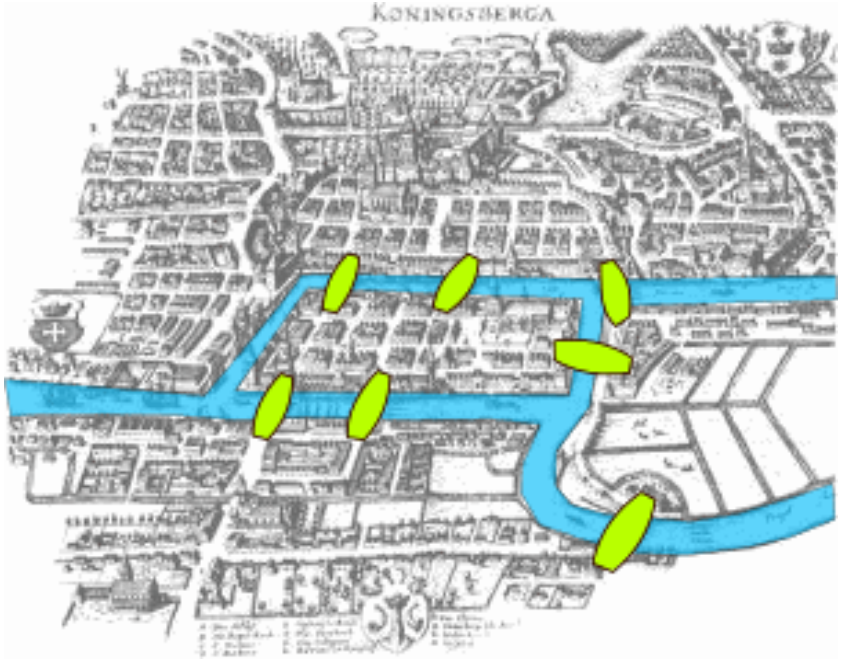
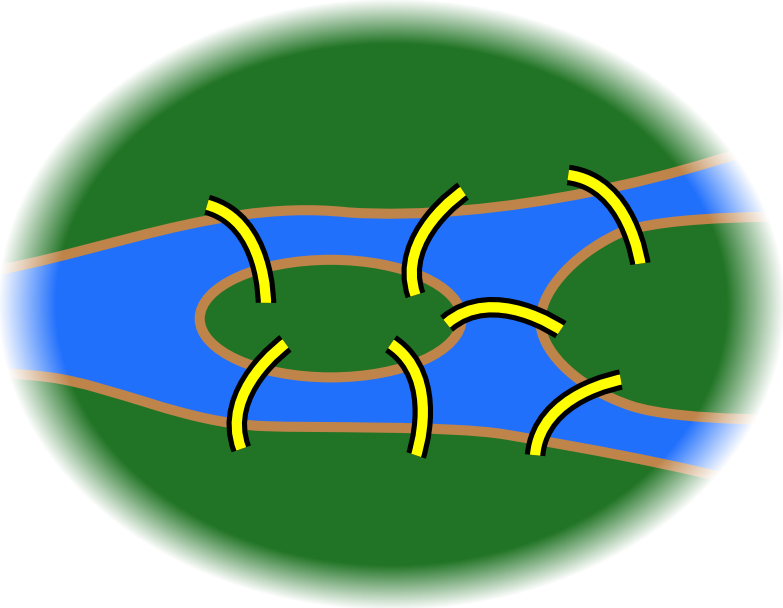


Seven bridges of Königsberg

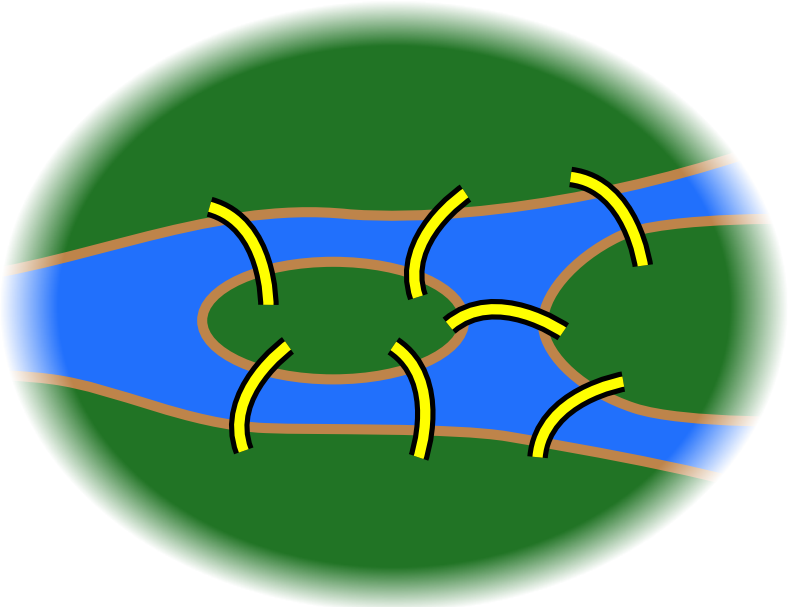
- Why not?
- Can it be shown, that such a walk doesn't exist?





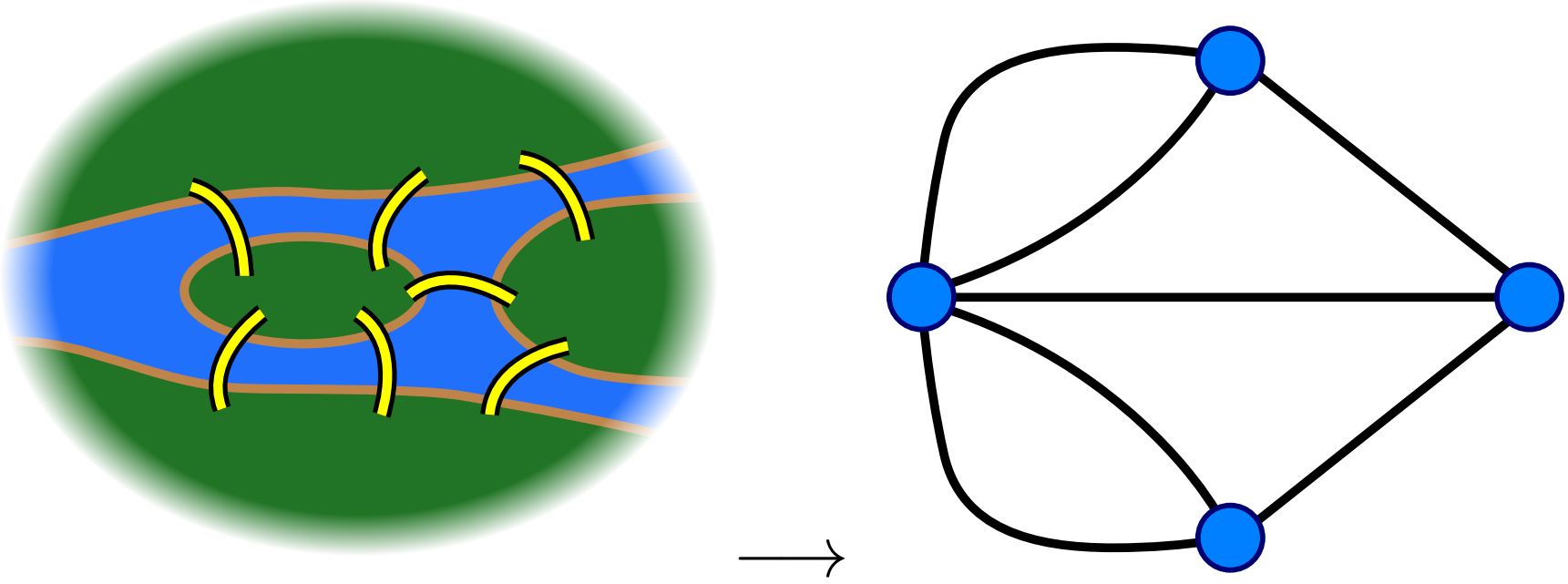
Seven bridges of Königsberg

- Why not?
- Can it be shown, that such a walk doesn't exist?



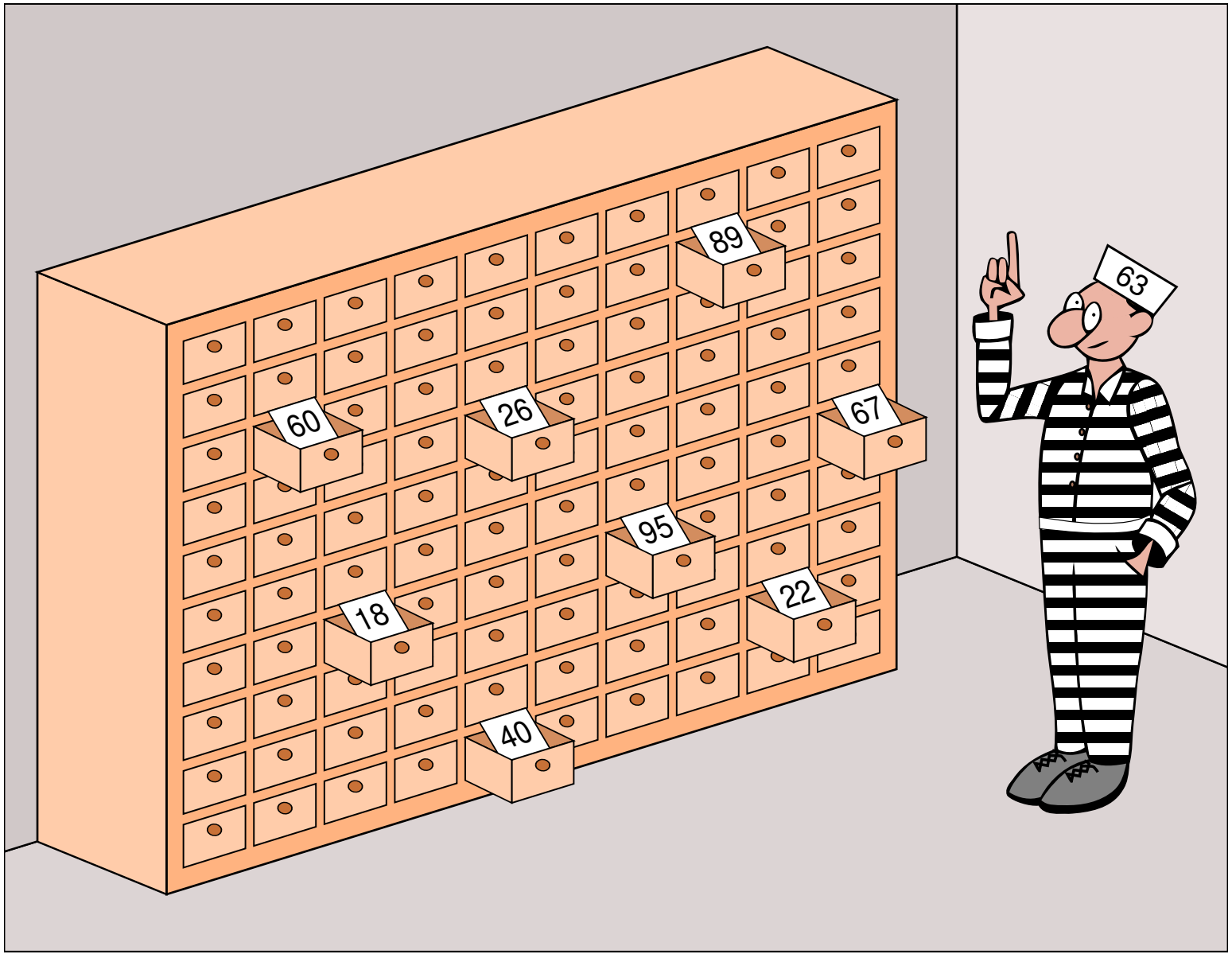
Seven bridges of Königsberg

- Why not?
- Can it be shown, that such a walk doesn't exist?



The 100 prisoners problem

100 prisoners problem



100 prisoners problem

- 100 prisoners, numbered 1-100

100 prisoners problem

- 100 prisoners, numbered 1-100
- 100 drawers, numbered 1-100

100 prisoners problem

- 100 prisoners, numbered 1-100
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- Every prisoner opens ≤ 50 drawers

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- Everyone is pardoned, if everyone finds their number

100 prisoners problem

- 100 prisoners, numbered 1-100
- 100 drawers, numbered 1-100
- Every prisoner opens ≤ 50 drawers
- No information exchange allowed during play
- Everyone is pardoned, if everyone finds their number
- If at least one prisoner fails, no one is pardoned

100 prisoners problem – Strategy?

- Logical prisoner: "We each open 50 drawers at random, there is no better strategy."

100 prisoners problem – Strategy?

- Logical prisoner: "We each open 50 drawers at random, there is no better strategy."
- Survival probability?

100 prisoners problem – Strategy?

- Mathematician prisoner: "We each open the drawer with our number and continue to open that drawer, which has the number we found in the previous drawer."

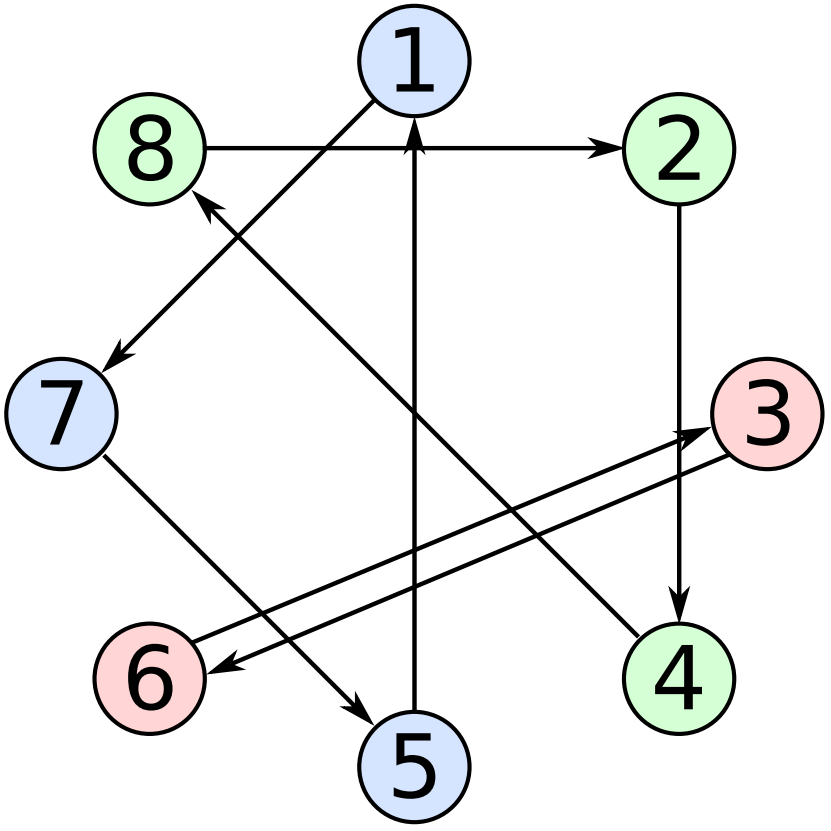
100 prisoners problem – Strategy?

- Mathematician prisoner: "We each open the drawer with our number and continue to open that drawer, which has the number we found in the previous drawer."
- Survival probability?

100 prisoners problem – Example 1

number of drawer	1	2	3	4	5	6	7	8
number of prisoner	7	4	6	8	1	3	5	2

100 prisoners problem – Example 1

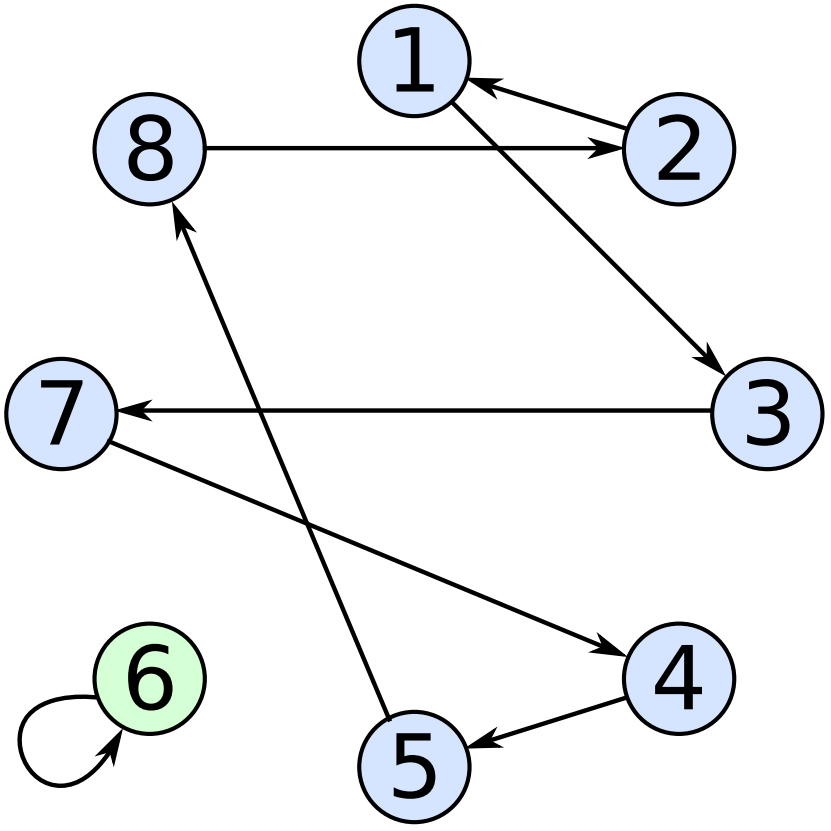


number of drawer	1	2	3	4	5	6	7	8
number of prisoner	7	4	6	8	1	3	5	2

100 prisoners problem – Example 2

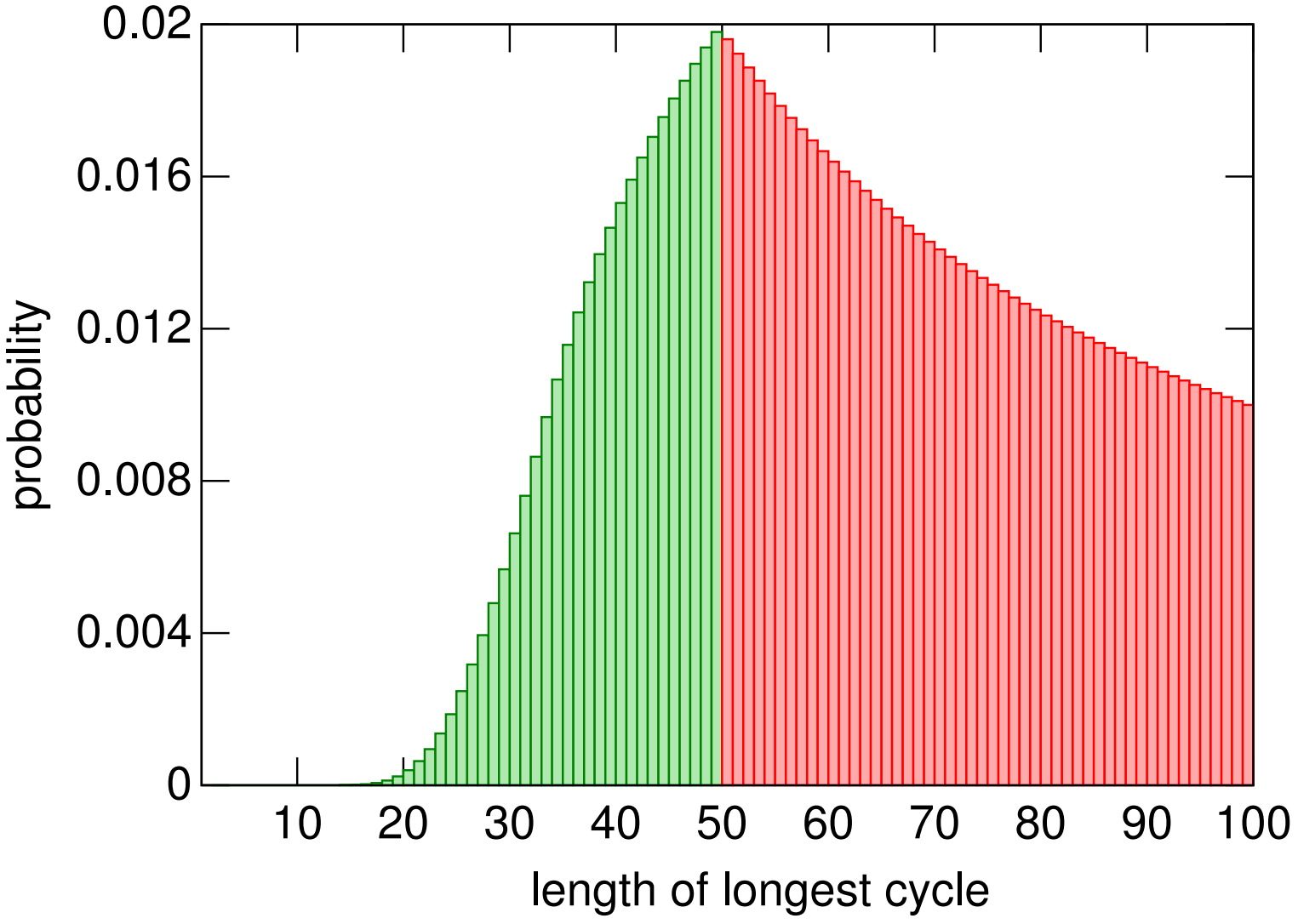
number of drawer	1	2	3	4	5	6	7	8
number of prisoner	3	1	7	5	8	6	4	2

100 prisoners problem – Example 2

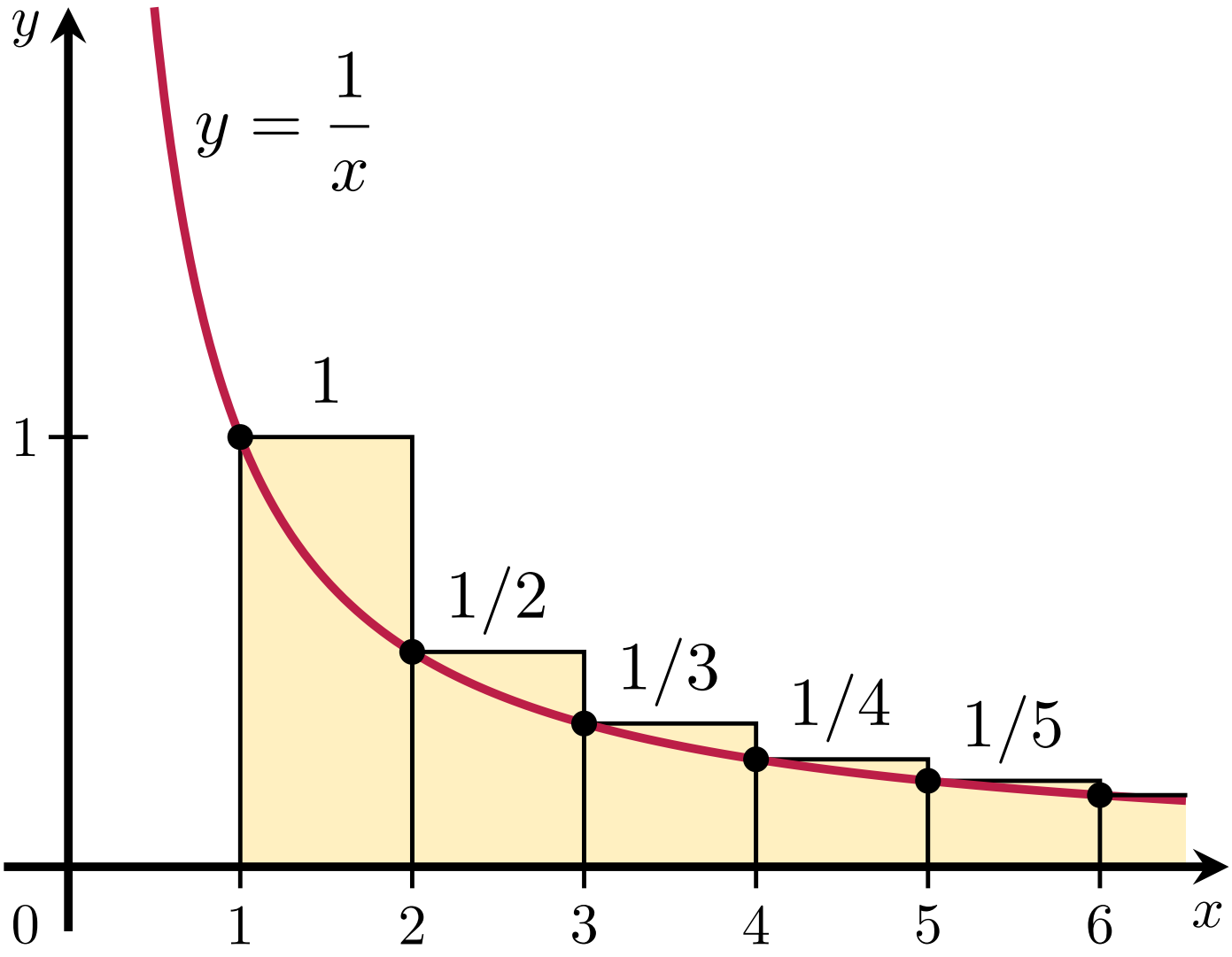


number of drawer	1	2	3	4	5	6	7	8
number of prisoner	3	1	7	5	8	6	4	2

Probability distribution of the length of the longest cycle of a random permutation



Harmonic numbers as an approximation of the area under a hyperbola



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– John von Neumann, 1947

Tips for long winter evenings

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- Introduction to Mathematical Thinking

  `https://www.coursera.org/course/maththink`

Tips for long winter evenings

- Introduction to Mathematical Thinking

  `https://www.coursera.org/course/maththink`

- Programming for Everybody (Python)

  `https://www.coursera.org/course/pythonlearn`

Thank you for your attention

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oskopek.com

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